Haphazard as a Factor in the Production of Tetrakaidecahedra*

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This paper, more fully presented than was possible in oral delivery and with added reference to subsequent publications, has been published in the American Journal of Botany, **30**: 74–81. Jan., 1943. There it is entitled "A Geometric Accounting for Diverse Shapes of 14–hedral Cells: the Transition from Dodecahedra to Tetrakaidecahedra." A summary of the discourse follows:

The study of cell shapes in compact parenchyma, or in similar unspecialized aggregates, has led to a series of surprises. (1) Such cells, instead of being rhombic dodecahedral products of surface tension, in reality have an average of between 13.5 and 14 facets,—usually close to 14. (2) The cells, though having an average of 14 facets, very rarely present the 14–hedral shapes deduced by Lord Kelvin as dividing space into uniform bodies of minimal surface. Even irregular or distorted approximations of those shapes, with 8 irregularly hexagonal facets and 6 quadrilaterals that are far from true squares, occur in less than 1 per cent. of the cells studied. (3) Compressed solids, such as shot of a given size, no longer controlled by surface tension, assume the same irregular cell-like shapes with the same average of close to 14 facets (Marvin). (4) Aggregations of soap bubbles of as nearly uniform size as they can be made, responding to surface tension, and free to glide over one another, do not assume the Kelvin shapes. With an average of 14 facets, they present a variety of cell-like forms (Matzke).

Confronted with this situation, the aggregation of geometrically perfect rhombic dodecahedra was considered anew. At six corners of each rhombic dodecahedron, when surrounded by others like it, six polyhedra meet at a mathematical point. Let two of them deviate a hair from meeting the other four at a point, and let the deviations throughout the mass occur in all directions at random, and the aggregation of rhombic dodecahedra becomes an assemblage of irregular shapes with an average of 14 facets. The shapes range from 12– to 18–hedra, and have an abundance of pentagonal facets. When all edges are more or less of the same length, these irregular polyhedra present many of the forms common to cells, bubbles in foam, and compressed shot. The average of close to 14–facets in a disarranged space-filling mass of bodies of similar size thus appears inevitable. The occasional occurrence of five or six cells chancing to meet at a point, or of four meeting along a line, would slightly reduce the average.

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Cell division extends the difference in the number of facets both above and below the afore-mentioned range of 12 to 18. Yet if the average plane of division is hexagonal (which would be expected when division bisects a cell rather than cuts off a corner) it will not affect the average of 14 facets. It causes a diversity in cell size, incompatible with a full realization of the Kelvin pattern. Yet if division occurs in a prevailing plane, it can orient the cells, and orientation makes possible an approach to Kelvin's orthic 14–hedron, which approximation is indubitably present in the oriented pith of *Eupatorium* and in similar tissue.

We conclude, therefore, that an average of 14 facets can be due to chance, or to tension, or a combination of both. The mathematical solution of the problem of dividing space into uniform bodies of least surface area and of maximum stability has been solved by Lord Kelvin's minimal 14–hedron (or its close approach,—his orthic 14–hedron). Since such diverse forms as the stellate 12–rayed cells of *Juncus,* and the prosenchymal tracheids of the pine with from 18 to 22 facets apiece, are accountable as derivatives of the Kelvin 14–hedron, as well as all the forms in cork and pith, it may properly be regarded as the typical shape of cells in masses. There is no rival uniform pattern. But it is only through absolute uniformity in size, precision in alignment, and the dominance of surface tension (3 factors at least) that a foam of minimal 14–hedra may be expected. These conditions have apparently not yet been realized in any cells or any froth. The typical shape thus remains a mathematical abstraction, whereas the actual shapes are coming to be well understood, and haphazard is a factor.

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